Vertically linked industries and the monocentric city

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Abstract

The vertical relations between the industry and more and more specialized services firms may explain the rise of suburban industrial clusters. In a urban economics model with two vertically linked sectors, the vertical disintegration liberates the location choice of the industry and the services. The industry clusters that need more land may leave the center. But when the industry is more linked to the services it agglomerates around the services, located at CBD.

Keywords: urban development, vertical linkages, spatial disintegration, suburbanization, firm location

JEL Classification: L23 - N90 - R12

1 Introduction

As J. Garreau [18] has pointed it out, more and more cities appear on the edge of the million towns. A metropolitan area is no longer just a big city. It's a miles long succession of Central Business Districts (CBD) and Industrial Clusters (IC). This implies that households have moved away from the center

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toward suburban areas. That means also important intra-urban movements of firms from the downtown area to the garden areas (Anas et alii [2], Glaeser and Kahn [19]). This paper focuses on the side of jobs and catches how the changes in the vertical relations between industry and services affect the location choices of the firms.

Actually, the spread of multipolar cities comes along with a strong spatial disintegration of the production. According to Fujita and Thisse [16], a spatial disintegration can be described as horizontal or vertical. It is horizontal if every unit of a multi-located firm host the same activities. It is vertical if all these units are specialized in different productions, such as in Los Angeles. There, Scott [27] notes that each sector tends to be clustered at a proper place.

The first type of organisation has lead to Christaller's type ([4]) of explanations. Looking at huge cities as continuous regional spaces, an entire strand of papers (beginning with Fujita and Krugman [11], but also Fujita and Mori [13] or Fujita *et al.* [12]) tries to rebuild a Christaller's type system of cities and clusters. Duranton and Puga [7] have studied the relations between size and specialisation within systems of cities and show that they are consistent with the hypothsesis of a horizontal spatial disintegration.

The second type relies more on industrial location choices and leads to von Thünen's ([29]) type of explanations. McLaren [25] has shown that firms have less and less interests in being strongly vertically linked as globalization increases. Following, Hansen [20] explains that along the industrial disintegration there is also a strong movement toward spatial disintegration. For example, industrial firms externalize their strategy and management to consulting groups. This reduces the cost of intra-firm distance that Ota and Fujita [26] put forward as the main centripetal force. As far as urban structure is concerned, the industrial disintegration makes multi-location of value chains within a city far more easier. Consulting groups minimize the inter-firm distance and might be clustered in the CBD to communicate with each other. Industrial firms tend to locate around the city because they need land more than contacts. The location of the factories of the automotive or the chemical industries in greater Paris tends to prove it. Thus, a vertical disintegration in the production may lead to a vertical spatial disintegration.

This paper tries to answer the question of whether two sectors vertically linked co-locate or not in a single city. Important forward linkages might induce co-location, but the activities can also chose different locations because they have different production functions and different uses of land. This model considers a two sectors economy. A quite simple way (but technically complex to achieve) to think about spatial disintegration is to split the firm into two units, a front-unit and a back-unit. Whereas an integrated firm has to aggregate the needs of all its activities, the break of a vertical linkage allows each sector to choose the most adapted location. Ota and Fujita [26] have built such a model where a firm has to decide where to locate a front and a back unit within a single city. They prove that the back unit may leave the center while the front unit remains in the CBD depending on the relative cost of intra-firm distance (between services and factories or front and back) and inter-firm distance (between services or front offices). Thus they have focused on integrated firms. As they point it in their conclusion, they neglect the direct interactions of back-units with other firms and households and they would need more general utility and production functions to account for them. Concerning the growing tendancy to externalization and vertical disintegration, this might be challenging.

This model uses vertically linked industries à la Krugman and Venables [23] in a urban economics way, as Abdel-Rahman and Fujita [1] have done. But their analysis remains incomplete regarding the question of spatial disintegration. They focus on the size of the city, rather than on its shape, and they assume that both types of firms are located in the CBD, while the concern here is to study the spatial vertical disintegration.

The remainder of this paper is organized as follows. In section 2, I present the model. It is a two sector model where the differences in the demand for land are crucial to understand the equilibrium. As the industry needs more land for its activities than services do, factories may find a suburban location more attractive. In order to simplify the solution of the model, the demand for land of the services firms is negligectible compared to that of the industry and the households. Hence, land is occupied by industry and households, and the need for land of the industry is a major centrifugal force. The links between services and industry keep jobs agglomerated and inbalance the centrifugal force. The solution depends on the equilibria on markets for intermediate services, for wages and for land.

In section 3, the model is solved considering only the existence of a monocentric city. Given this configuration and the equilibrium on the different markets, we work out the conditions on the parameters that make it possible for the configuration to be an equilibrium (in the sense of Nash equilibrium configuration defined by Ota and Fujita [26]).

In section 4, I explore the results that derive from the model. Especially, we look at how and why the monocentric city is sustainable when parameters change.

2 The model

The city is supposed to be open, linear, with absentee landlords that receive the land rent R. It is supposed to be centered in 0 with an endogenous population L. In addition, the model assumes a monocentric pattern.

The core of the model is the different needs for land and communication of the two sectors of the economy. Industry needs land and externalizes its need for information, whereas services firms do not need any land but improve their productivity through contacts with other firms. Therefore, the services firms face agglomeration externalities and do not have any demand for land while the industry is a constant return sector and uses land.

Households are indifferent between the two types of jobs (in industry and services). All the services are sold to the industrial firm, and there is a Chamberlin type of competition on the market for intermediate services. Each of the N services firms hires L_1 workers (N is endogenous and depends on the tightness of the vertical linkages). Industrial firms have a overall demand for land, S. They also buy aggregate services (Q), labor (L_2) and they export their production Y at a fixed price r. There is no agglomeration externality in the industry.

The transportation costs in the city, t, affect households and services. They are additive. They only apply to the land occupied by households, while industrial areas are supposed to be free from transportation costs. This is not a rough assumption, since it only means that the firms are indifferent between all the possible places in the cluster and that the households can cross the cluster without entering it[1]. An immediate consequence is that the IC can be considered as a single point, b, and that industrial firms are concentrated without assuming externalities. Because all the firms are identical

[1] If we assume that there is a freeway through the cluster, the households can cross it far more quickly than a residential area. Also, as opposed to the residential areas, the transportation network within the cluster is adapted to the needs of the industry. Thus within the cluster both the transportation costs of persons and goods are neglectible compared with that of a residential area. Hence we consider here that the cost of entering or crossing a cluster is nul.

and because the differences between two possible locations only rely on the transportation costs, the best location will be chosen simultaneously by all the firms and there will be a new cluster[2].

As in most papers in urban economics, there are several possible equilibria depending on the location of services and of the industry, depending also on the exclusive use or on the mixed use of land, etc. In what follows, the question is only that of the existence of a monocentric city. In the sense of Nash, the monocentric equilibrium can exist only if, given its central location, no firm finds an incentive to relocate.

2.1 The households

Households work indifferently in the two sectors and must choose between the consumption of land (size l) and that of an imported composite good, z, whose price is normalized to 1 and is the numeraire of the economy. Since they work indifferently in one sector or the other, the L households manage first to choose their place of work. They decide the place where they want to live only afterwards, by maximizing their utility $U = U(l, z)$ considering the set of their consumption of both land and composite good. There is only one worker per household and the only income the households get are their wages. We assume that the composite good is imported at a uniform price that includes the transportation cost. Therefore, with l normalized to 1 and $U = U^* = U(z^*)$, the city being open, the budget constraint of a household living in x and working at x_w is

$$W(x_w) = R(x) + z^* + td(x, x_w),$$

with $W(x_w)$ the wage earned if the firm is located in x_w, $R(x)$ being the land rent at x, $d(x, x_w) = |x_w - x|$ being the commuting distance, and t being the transportation cost.

Each household aims at maximizing its utility with regard to this constraint. Given an exogenous level of land use normalized to one, each household will have to choose x so as to maximize its consumption of composite good. Hence, when the place is occupied by a household, the land rent must be equal to the bid rent of the household $\Psi(x)$ which also depends on x_w

[2] We assume here that all the firms decide to move at the same time and without any coordination. All firms being the same, a situation in which only a non neglectible part of the industry is moving is impossible.

(Fujita [8]), with $W(x, x_w) = W(x_w) - t|x_w - x|$ the wage free from transportation cost.

There are $L(Y)$ households and the overall population of the city depends on the one hand on the production of the industry Y and the employment in services induced with $L(Y) = L_2(Y) + N(Y)L_1$ and on the other hand on the equilibrium on the land market. If a higher production means a bigger population hence a higher rent at the CBD, this tightens the budget constraint of the industry and might reduce the production and the population of the city.

The equilibrium wage depends on the relative locations of the employment centers (IC and CBD). Considering the specific case of the monocentric city, $x_w = 0$, the curve of the wages free from transportation costs $W(x, 0)$ is strictly decreasing. As a household locates near the edge of the city it costs more and more to reach the center. On the edge and once the household has paid for z^* and the transportation, what remains just equals the cost of opportunity of the land, usually its agricultural productivity here equal to zero. Thus $W(0) - t\frac{L(Y)}{2} = z^*$. The bid rent of a household in the monocentric case can be written

$$\Psi(x, Y) = W(x, Y) - z^* = t\left|\frac{L(Y)}{2} - x\right|, \tag{1}$$

and, at the optimum of industrial production, $\Psi(0) = W(0) - z^* = t\frac{L}{2} = \Phi(0)$ must be verified.

2.2 Services

Services firms are in a Dixit-Stiglitz competition to sell their products to industrial firms. Each services firm finds an advantage to be located near the others (Lucas [24], Anas *et al.* [2]). In the case of a monocentric city, if a firm sets up near the CBD (which is supposed to be located at a with either $a = 0$ or $a = b$), it will benefit from the proximity of its competitors. With k being a fixed cost and $\delta > 0$ setting the importance of the externalities, the production of the i^{th} services firm is

$$q_i = f(L_{1,i}, a, x_i) = \left(1 + e^{-\delta|x_i - a|}\right)L_{1,i} - k.$$

Using the exponential form, the higher δ, the lower the externalities. When all services firms are clustered, the externalities are maximum. When every firm locates far away from the others the effect of externalities on the production of services is nul. It occurs when x_i tends to be infinite. Note that in that case, the services firms are less productive than the industry because they have to overcome the fixed costs ($e^{-\delta|x_i-a|}L_{1,i} < k$).

The price of the services includes the delivery to the industry, located at b. The profit of a services firm is then

$$\pi_{1,i}(x_i) = p_i q_i - [t|b - x_i| + W(x_i)] \frac{q_i + k}{1 + e^{-\delta|x_i-a|}}. \tag{2}$$

An increase in the fixed cost will affect the profit of the firm, as well as an increase in δ, for $x_i \neq a$. An increase in the transportation cost directly affects the profit of the services firm since it increases the delivery cost. By the way, it also increases the wages through (1) and this second effect reinforces the main one.

The equilibrium on the market for intermediate services can then be determined. The production of the services firm is exclusively dedicated to the industry. Given the preference for variety of the industry and the oligopolistic competition on the services side, the industry addresses to each services firm a demand that depends on the prices index P, the overall demand Q, p_i the price of the i^{th} variety and the substitution rate among all the services, σ. With q_i being the amount of services sold by the i^{th} of the N firms of services, the overall services consumption is

$$Q = \left(\sum_{i=1}^{N} q_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where σ is the substitution elasticity between the services. Here $\sigma > 1$, and as it increases, the services are more and more substitutes with each others. Now, $\rho = \frac{\sigma-1}{\sigma}$ stands for the preference for variety ($0 < \rho < 1$) which is usually a key parameter in NEG models. The higher the substitution elasticity the lower the preference for variety (ρ high, or σ low), and the higher σ, the less the services are specialized.

Minimizing the cost of the aggregate services Q, this gives

$$\sum_{i=1}^{N} p_i q_i = \left(\sum_{i=1}^{N} p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} Q.$$

From now on $P = \left(\sum_{i=1}^{N} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is the prices index of the aggregate services.

This leads to the following production constraint for each services firm,

$$q_i = \frac{p_i^{-\sigma}}{P^{-\sigma}} Q. \tag{3}$$

The i^{th} firm maximizes its profit (2) with regard to the price of its services p_i and considering $G(b) = PQ$, the budget of the industry dedicated to services:

$$\max_{p_i} \pi_{1,i} \quad \text{s.c.} \quad q_i = p_i^{-\sigma} G(b)^{\sigma} Q^{\sigma-1}.$$

If all the firms are agglomerated at a, $x_i = a$ $(\forall i)$. Since they produce the same amount of services and sell them at the same price, the equilibrium price $p_i = \bar{p}$ $(\forall i)$ can be found out and the equilibrium quantity $q_i = \bar{q}$ $(\forall i)$ can be derived:

$$\bar{p} = \frac{\sigma}{\sigma-1} \frac{t|b-a| + W(a)}{2} \tag{4}$$

$$\bar{q} = (\sigma-1) k. \tag{5}$$

We verify the classical result according to which the services sold by each firm at equilibrium do not depend on the size of the industrial production. They are just related to the importance of the fixed costs and that of the preference for variety. If the production increases, there just will be an increase in the number of firms. With the two preceeding expressions, the budget constraint of the industry can be rewritten

$$N\bar{p}\bar{q} = G(b), \tag{6}$$

that is to say

$$N = \frac{2}{\sigma k} \frac{G(b)}{t|b-a| + W(a)}. \tag{7}$$

Now, the agglomerated case will be an equilibrium only if no services firm finds it profitable to relocate near the industry. Considering the case of a monocentric city, this means that with $a = 0$, no firm leaves the center for the periphery in order to benefit from lower transportation costs. The binomial choice to relocate near the industry (b) or to remain in the center will thus depend on the relative level of the transportation costs (centrifugal) and of the externalities (centripetal). A firm relocates as soon as $\pi_{1,i}(b) > \pi_{1,i}(a)$. Using (4) and (5),

$$\begin{aligned} p_i(b) &= \frac{\sigma}{\sigma-1}\frac{W(b)}{1+e^{-\delta|b-a|}} \\ q_i(b) &= \left(1+e^{-\delta|b-a|}\right)^{-\sigma} 2^{1-\sigma}(\sigma-1)k \end{aligned}$$

and the profit of a relocated firm (2) can be rewritten

$$\pi_{1,i}(b) = \frac{W(b)}{1+e^{-\delta|b-a|}}.A$$

With $A = \left(1+e^{-\delta|b-a|}\right)^{-\sigma} 2^{1-\sigma}k + k$, $\pi_{1,i}(b) < \pi_{1,i}(a)$ if $A < 0$. Knowing $b-a = \frac{L}{2}$, and because $\sigma > 1 \Rightarrow 2^{\sigma-1} > 1$, we reach the following condition on δ:

$$\delta > -2\frac{\ln\left(2^{\sigma-1}-1\right)}{L} = \kappa.$$

Note that as long as $\sigma > 2$, the condition is verified as soon as there are externalities among services ($\delta > 0$). Now, with $\frac{\partial\kappa}{\partial\sigma} = -\frac{2^{\sigma-1}(\sigma-1)}{L(2^{\sigma-1}-1)} < 0$, a higher specialisation of services (a decrease in σ, especially if $\sigma < 2$) means that the externalities must be stronger to prevent services firms from relocation (κ increases). On the contrary, an increase in the population lowers the constraint ($\frac{\partial\kappa}{\partial L} < 0$).

The following considers that the services firms remain gathered in the center, i.e. that $\delta > \kappa$ is always verified.

2.3 The industry

The city exports an industrial good, produced by n industrial firms (n sufficiently high). We assume the competition between the industrial firms to be pure: they all use the same process and there are no externalities that influence the location of the industrial firms. As there are no transportation cost within the IC the location of a firm has no effect on the commuting cost. Hence it has no influence on the wage curve. By the way, the location of a firm can be considered as a single point from the land rent pattern point of view. Because all the firms are identical and because the location of one firm has no influence on the land pattern, all the firm are located at the same place and all the prices will be set as for an IC located at a single point b.

As a whole, the industry produces

$$Y = \min\left\{\frac{S}{\bar{s}}, Q^{\lambda} L_2^{1-\lambda}\right\}, \tag{8}$$

where $\bar{s}$ stands for the land productivity (from the industry point of view) and λ characterizes how much the industrial production is externalized to the services firms.

The industry profit is then

$$\Pi_2(b) = rY - S\Phi(b) - W(b)\,L_2 - \sum_{i=1}^{N} p_i q_i$$

The part of the budget the industrial firms dedicate to both wages and intermediate services $T(b) = r - \bar{s}\Phi(b)$ clearly depends on the location of the cluster. A central location increases the price of the land. Now we need to consider the intermediate consumption as a whole. With $\sum_{i=1}^{N} p_i q_i = PQ$, the industrial sector maximizes its profits with regard to the technical constraint (8). Thus the following solution comes naturally,

$$\begin{aligned} S &= \bar{s}Y \\ L_2 &= \frac{1-\lambda}{W(b)} T(b)\,Y \\ Q &= \frac{\lambda}{P} T(b)\,Y. \end{aligned} \tag{9}$$

and $\lambda T(b)\,Y = G(b)$. Hence the classical result, $T(b) = \left(\frac{W(b)}{1-\lambda}\right)^{1-\lambda}\left(\frac{P}{\lambda}\right)^{\lambda}$. Profit must be null at equilibrium and the industrial land rent can be expressed as what is left to the industry once services and labor have been bought. Using all the preceding results on S, L_2, Q and $T(b)$,

$$\Phi(b) = \frac{r}{\bar{s}} - \frac{1}{\bar{s}}\left(\frac{W(b)}{1-\lambda}\right)^{1-\lambda}\left(\frac{P}{\lambda}\right)^{\lambda}. \tag{10}$$

This corresponds to the industry willingness to pay for land. For the land use pattern to be an equilibrium, $\Phi(.)$ must be equal to the land rent at the places occupied by the IC. It depends on the earnings of the industry minus its costs, both labor costs and services costs. With (10), (6) and (7) we find that if the conditions on the wages are verified so that $\Phi(0) = \Psi(0, Y)$,

$$\Phi(0) = \frac{r}{\bar{s}} - \frac{z^* + \frac{tL(Y)}{2}}{\bar{s}} \left(\frac{1}{1-\lambda}\right)^{1-\lambda} \left(\frac{L(Y)}{k}\right)^{\frac{\lambda}{1-\sigma}} \left(\frac{2\lambda}{\sigma}\right)^{\frac{\sigma\lambda}{1-\sigma}} (\sigma - 1)^{-\lambda}.$$

In addition, it is assumed that there is no unemployment in the city and the population works either for the industrial firms or for the services firms. Given the population and the needs of both industry and services, we will be able to split the L workers into L_2 industry workers and NL_1 services workers. The way the split is completed will then determine the production exported by the city Y.

We calculate the number of industry workers using (6) in $\Pi_2(b)$ and considering that $\Pi_2(b)$ is nul at equilibrium:

$$L_2 = \frac{(1-\lambda) YT(b)}{W(b)}. \tag{11}$$

Now, as we know

$$NL_1 = \frac{\lambda T(b) Y}{t|b-a| + W(a)}, \tag{12}$$

we can express Y as directly linked with the size of the population and the economic structure (through σ and λ). Using the two preceding equations in the definition of $L(Y)$, we find that the production depends on the size of the population and on the split between industrial workers and services workers.

$$Y = \frac{L(Y)}{\left(\frac{1-\lambda}{W(b)} + \frac{\lambda}{t|b-a|+W(a)}\right) T(b)}.$$

As a specific wage is associated to each place, a relocation of the CBD or the IC will modify the wage pattern and change the entire production of the city. A relocation of one or both of the clusters will have a price effect through the relative wages and an income effect through the budget constraint.

Lemma 1 *In the case of a monocentric city, the cost curves of the industry are homothetical.*

When the city is monocentric,$a = b$ and $W(b) = t|b-a| + W(a)$. Thus the production can be written $Y = \frac{LW(b)}{T(b)}$. In that case, the price effect impacts only the level of the production. If it is not monocentric, then $t|b-a|$ is positive and there is a substitution effect between wages and services.

The distance between the IC and the CBD will also influence the delivery price of services thus modifying the production at equilibrium. Also, these locations have an influence through the transportation costs. The congestion (high transportation costs) has two effects on the city. First it lowers the earnings of the households, and second it increases the delivery price of services.

There is no unemployment in the city so the population immediately changes to fit the level required for production needs. Once the production is set, the population adapts itself.

At equilibrium, $\Phi(0) = \Psi(0,0,Y) = R(0)$ and $T(0)$ can be derived from the preceeding expressions. Thus, the production of the monocentric city considering the population of the city is

$$Y = B.L^{\frac{\sigma+\lambda-1}{\sigma-1}},$$

with $B = (1-\lambda)^{1-\lambda} \left(\frac{2\lambda}{\sigma}\right)^{\frac{\sigma\lambda}{\sigma-1}} k^{\frac{-\lambda}{\sigma-1}} (\sigma-1)^{\lambda}$.

Remark 1 *An increase in the size of the city will raise the production. But this happens only if the industry externalizes enough services to highly specialized firms (*$\lambda > \sigma - 1$*)*

Proof: With $0 < \lambda < 1$ and $\sigma > 1$, we know $\frac{\sigma+\lambda-1}{\sigma-1} > 1$ and $B > 0$. Thus, $\frac{\partial Y}{\partial L} > 0$ from the expression above.

Remark 2 *The higher the specialization of services, the higher the industrial production*

Proof: In the expression above, the production is linked to the preference for variety of the industry through the productivity of the services firms and the respective level of employment in both sectors. It comes that

$$\frac{\partial Y}{\partial \sigma} = e^{\lambda \ln(\sigma-1)} \frac{e^{\frac{\sigma+\lambda-1}{\sigma-1}\ln L} e^{(1-\lambda)\ln(1-\lambda)} e^{\frac{\sigma\lambda}{\sigma-1}\ln 2\lambda}}{e^{\frac{\sigma\lambda}{\sigma-1}} e^{\frac{\lambda}{\sigma-1}\ln k}} \frac{\lambda}{(\sigma-1)^2} \ln \frac{\sigma k}{2\lambda L},$$

which sign follows that of $\ln \frac{\sigma k}{2\lambda L}$. Hence, using (7), $\frac{\partial Y}{\partial \sigma} > 0 \Leftrightarrow N < 1$, and if the preference for variety increases (σ decreases) the production increases also.

3 Solution

The relation between the production Y and the size of the population makes it clear that if the households and the industry compete for land in the city, their bid rents are linked via the wages. The wages determine the budget constraint of the households and their willingness to pay for land at each location. The wages also enter in the production function of the industry firm either directly or via the prices for services. Thus the level of production and the level of land consumption as well as the willingness to pay for land derive from the wages.

For a monocentric city to be at equilibrium, the bid rent of the households must be equal to that of the firms in the center

The land rent of the households both depends on the wages and on the size of the city. Because there are no transportation costs in the IC, the size of the city is given once the population as been fixed. Then, the transportation costs but and the price of the industrial good will only have nominal effects. An increase in r or a decrease in t will increase the land rent of the industry. Hence, we reach a minimum condition on r, such that the city exists

$$\Psi(0) = \Phi(0) \Leftrightarrow r = \frac{\bar{s}t}{2} + \frac{2z^* + tL}{2L^\sigma}\left(\frac{L}{1-\lambda}\right)^{1-\lambda}\left(\frac{(2\lambda\sigma)^{\frac{\sigma}{\sigma-1}}}{\sigma-1}k^{\frac{1}{\sigma-1}}\right)^{\lambda}.$$

The optimal size of the city can thus be derived from the preceeding expression. With $f(0) = \Phi(0) - \Psi(0)$, we just consider that at equilibrium the city is such that $\frac{\partial f}{\partial L} = 0$ and that $f(0) = 0$.

We can not analytically derive the optimal size but numerical simulations for common values of parameters (here $z = 1; r = 15; \bar{s} = 1.7; k = 1.3$ and $t = 0.2$ and $\sigma = 2.1$[3]) ensure we always reach positive solutions.

As it appears in Figure 1, the optimal size is first increasing then decreasing as the externalization rate λ increases. Quite intuitively, the size of the city depends on the main centrifugal force of the model: it is strictly decreasing with t. It is also decreasing with the specialisation of services (increases with σ - fig 1).

[3] The ratio z/r defines the type of industrial production. In that case, the good corresponds to almost 3 months of wages. Other values have been tested for k and $\overline{s}$ and other simulation have been held on wider ranges of parameters, $\lambda \in [0;1]$, $\sigma \in [1.1;3.6]$ and $t \in [0.01;0.35]$. The results are plotted at http://gilli.ensae.net/These/Chapt7.pdf, pages 264 sq.

<Insert Figure 1>

Now the city remains monocentric as long as the industrial firms do not find any incentive to relocate. There are n atomistic industrial firms. The relocation of a single firm will not affect the various equilibria of the economy, but it will change the program of that single firm. Nevertheless, because all the firms are identical, they will all take the same decision. If the decision of one single firm has no consequence on the city, that of the entire IC will. It can even downgrade the earnings at the new location because a shift of the IC will change the commuting cost of the households and thus the wage curve and the land rent. Thus the new location might not be an equilibrium. The following focuses on the existence of the monocentric equilibrium. The existence of an alternative equilibrium at the periphery when the center is not stable is not studied here.

One industrial firm located at x when the IC is at b will have a specific demand for labor l_2 and a specific demand for services q given the prices of both the labor ($W(x)$ instead of $W(0)$) and the services ($\bar{p} + tx$ instead of $\bar{p}$). Note that in the remainder, the small letters characterize the values for a single industrial firm. The program of a single industrial firm is

$$\pi_2(x) = ry - \bar{s}y\Phi(x) - W(x)\,l_2 - N(\bar{p} + tx)\,q$$

and it is optimized with regard to the technical constraint $y = l_2^{1-\lambda} q^{\lambda}$. Solving the program, we reach the following condition according to which the firm will not find any interest to leave the center : $C(x, 0) = \pi_2(x) - \pi_2(0) < 0$. Now,

$$C(x, 0) = \bar{s}yxt + tl_2x - Ntqx.$$

Given the level of externalization (λ), both wages and services affect only the profit through their relative prices. The effect of a relocation depends on an income effect and on a price effect. At x, the earnings and the land rent fix the budget constraint. If the cost curves of the industry are homothetical, then a relocation does not have any impact on the relative consumption of inputs. The global effect will thus depend linearly on the distance between the new location and the IC. The monocentric pattern belongs to this category (*see:* Lemma 1) and the condition linearly depends on x and is monotonous. Using (11) with (6) and (4) and l_2 and q from the solution of the new program,

$C(x',0)=0 \Leftrightarrow$

$$x'=\frac{-\bar{s}W(0)-T(0)+T(0)(1+2\rho)\lambda}{\left(s+\frac{T(0)}{W(0)}\right)2t\rho-4\rho+(st-2)(1+2\rho)\lambda}.$$

If $x'<0$ the monocentric city will be an equilibrium because no industry firm finds any interest in leaving the center. If $0<x'<L/2$, the IC will leave the center and relocate on the edge of the city where its profit will be maximum. If $x'>L/2$ and considering that the transportation costs are null within the IC, the monocentric city is also an equilibrium.

The relative locations of the IC and the CBD of course depend on the preference for variety of the industry and of its level of texternalization. All things equal, a higher λ means that the industry will use more services than employees. As the wages are a centrifugal force whereas services remain a centripetal force, a monocentric-agglomerated city will be more likely to exist when the production is externalized. But when the economy is mostly industrial (λ very low) the productivity of the economy is low enough so that the wages and the land rent of the households can not compete for land with the IC at the center. A central location for the IC might also be possible in that case.

The impact of σ is mostly that on the price index for services. A higher specialisation (lower σ) highers the prices for services and gives more importance to the centripetal force.

4 Results

4.1 Simulations

When a firm relocates, its land rent, its wages, as well as the price it pays for the intermediate services do change and it is possible to determine which point maximizes the profit. The condition being linear on $[0;x)$, this point is necessarily in the center or on the frontier of the city. Once the size of the city has been determined, the sign of the condition on the frontier is the threshold: if it is positive, the IC might relocates and if it is negative the monocentric city exists and is an equilibirum.

At first, with $\sigma=2$, the effect of t and λ on the constraint shows that the relocation is more and more profitable as long as lambda increases (Figure

2), because of the effect of the rise of services on productivity. After a critical value (around $\lambda = 0.6$), the IC depends so much upon services that even if the center is not affordable, the centripetal force due to the delivery of the services is fearcer and the city remains monocentric (fig 2).

<Insert Figure 2>

The same analysis can be held considering the impact of σ on the constraint $C(.)$. When there is a high elasticity of substitution among services, or a low specialisation, $C(.)$ is mostly negative and the condition of existence is easy to clear. But when the specialisation rises, there are more and more incentives to relocate for the industry and the condition for the existence of a monocentric city is not met anymore. for very high levels of specialization (around $\sigma = 1.2$) there is even a break : the condition is cleared for very low levels of externalisation (around $\lambda = 0.2$), but in the ranges of parameters considered above the high specialisation of services leads to a re-agglomeration of the industry around the services and the monocentric city exists (fig. 3).

<Insert Figure 3>

For each value of transportation cost t and based on the sign of the condition, it is possible to determine the entire set of $(\sigma; \lambda)$ such that the $C(x) < 0$, that is such that the monocentric city exists and is an equilibrium (fig. 4). It is also possible to plot the curve $\frac{\partial C(x,O)}{\partial \lambda} = 0$ which differentiates two kinds of trends, those when the centrifugal forces are too weak to threaten the moncentric equilibrium (low λ) and those when the centripetal forces are strong enough to compensate the growing importance of centrifugal forces (high λ) (fig 5).

<Insert Figure 4, 5>

Remark 3 *Higher transportation costs mean more spatial desintegration and lead to a sprawl of the industry*

This result could seem a bit paradoxical. But the effects of transportation costs in the model are twice. Higher transportation costs mean higher centripetal force due to the price of services. And it also means a higher land rent which pushes industry away from the center. Since this second effect overcomes the first one, higher transportation costs will weaken a monocentric city. Note that for low values of λ (low externalisation) an increase in the transportation has barely any effect since the centripetal force is already weak. On the contrary, a decrease in the transportation cost has a strong effect on the cities with high externalisation rates as it can be seen on figure 6: a lower transportation cost immediatly reinforces the relative effect of the price of services compared to that of the land rent and enables re-agglomeration of services and industry.

<Insert Figure 6>

4.2 Comments

The relative locations of the IC and the CBD highly depend both on the preference for variety of the industry firms and on the externalization of their production toward the services firms.

All things equal, a higher λ means that the industry firms externalise more their production so thy substitute labor for services. The wages being a centrifugal force in the model while services are a centripetal one, the more externalized the economy, the more likely a monocentric configuration. But when the city is a quasi pure industrial one the land rent is very low. Hence the central location can also be attractive for low values of λ.

The preference for variety of the industry mostly has an influence through the price index of the services. Namely, an increase in the preference for variety will increase the price index and the relative importance of the centripetal force. Thus, for low values of σ when the services are specialized, the monocentric equilibrium will be more sustainable.

Also a change in the function of production changes the overall production and then the budget constraint. The consequences of this income effect highly depend on the cross relations between externalization and specialization. Now, all things equal an increase in the externalization of the production to the services firms increases the production of the city. For low values of λ, these cross effects will be highly negligeable. But as the industry turns more and more toward services firms, the importance of the income effect will increase.

Definition 1 *A city is an* industrial city *when industry and non-specialized services are both located in the center.*

It is a modern city *when the industry is located around the city and partly externalizes its production to services firms.*

The post-modern city *is characterized by the co-location of the IC and the CBD. The production is highly externalized to specialized services firms.*

Remark 4 *There are three types of cities depending on the specialization of the services and externalization of the production. In the first one, when both are low, the monocentric equilibrium exists. An increase in any of the parameters may lead to the the second type, when industry firms tend to leave the center and to locate around the city. Finaly the moncentric city is an equilibrium again with high externalisation and high specialisation because the IC relocates close to the CBD.*

Considering the evolution of the condition as specialization and externalization change, three regimes of cities appear (fig. 7).

<Insert Figure 7>

The first regime corresponds to the industrial city (**1**), when the city is mostly industrial and the services are not specialized. In that traditional city, the industry is located around the center and the production is highly integrated. If the city is mostly industrial, the land rent is not very high. If the preference for variety is also low, then it makes it easier for an industry firm to remain in the center. But if the industry externalizes more of its production to services firms or if the services are getting more specialized, the production and the prices will rise in the city. The monocentric city could not remain an equilibrium mostly because of the income effect. Historically, this transition corresponds to the roaring twenties in the USA and the fifties in Europe, the turning point being the development of the mass production.

The second regime is that of the modern city (**2**). The industry leaves the center and settles around the city, looking for land and accessibility. Due to both an emerging disintegration of the production and the growing importance of more specialized services, the production of the city increases. Without any size effect (the population remains constant in our model) but via the income effects on wages the land rent increases in return. This leads to a city where the services are gathered in the CBD while the IC locates on the edge where land is more affordable. J.Garreau depicts it well when he deals with the case of Detroit. He writes,

> "... it offered far more land than the old down-town both for expansion — and parking. Right there, in New Center, immediately after World War 1, Edge City was probably born. Henry Ford's company followed suit. When he switched production from the Model T to the Model A in 1928, he also switched his *factory* location from Highland Park (...) to the plains of Dearborn."

But that dynamic can reach a stage when the centripetal force of the services overpasses the centrifugal forces of both wages and land rent. Then the industry firms do not find anymore incentives to leave the CBD. Once again an increase in the specialization of the services acts the same way. As the industry externalizes a growing part of its production to the services firms, more and more substitution effects influence the production of the city and the profit of the industry firms.

The third regime is the post-modern city (**3**). The industries are back in the city. But this come back is not driven by a recovered productivity that could increase the land rent the firms offer. As it externalizes more and more of its production to the services firms the industry reinforces the centripetal forces that links the IC and the CBD. The growing importance of the services increases the centripetal force that finaly overpasses the centrifugal effects of both wages and land rent. It leads to integrated clusters around the CBD as in lot of contemporary American cities.

For a low degree of specialization (namely, σ high), the multipolar city may never exist. In such a case, the IC can afford a central location because the land rent never exceeds its willingness to pay. For low or medium externalization rates, it can compete with the households for a central location. For high externalization rates, it is fixed at the center by the services and the city might remain monocentric since the services have always remained in the center.

Note that the comments focus on the way externalization and specialization interact. Of course, the other parameters influence the evolution as well. An increasing transportation cost, for example, set a lot of parameters in favour of regime 2. Namely, as transport is costly suburban IC are more likely to appear which is quite intuitive as regard the urban economics litterature.

5 Conclusion

In the huge metropolitan areas there is a joint movement toward suburbanization of the industry and specialization of the services. The model shown in this paper explains the consequences of both these evolutions on urban development. It traces the evolution of the city back from its industrial age to the post-modern figure of Los Angeles through the evolution of the vertical relations between industry and services.

The vertical disintegration of the industry makes it possible for the different sectors to chose their location according to their own criteria. Hence, even if there remains a vertical relation between separated firms, there may be a spatial disintegration in the urban space. In the model as in most of the urban economics models, the multipolarization of the city comes from a comparison between the centrifugal and the centripetal forces. The transportation cost plays an important role. As usual, it is a centrifugal force since it leads to an increase in the land rent and the wages paid at the center. As industry needs land, industry might leave the center. But it enters also as the cost of the vertical relations linking the industry and the services. As such it is a centripetal force which increases when the economy is more and more services oriented.

The joint introduction of the vertical linkages and of the specific demands for land leads to an original approach of the urban development history. The industrial city is a city where industry is mostly integrated and services are not extremely specialized. It corresponds to the first regime, when the monocentricity is an equilibrium. In the first part of the twentieth century the mass production system lead to the modern city where the CBD remains in the center while the IC locates around the city. This regime has generalized after WW2. But then, as both a growing externalization and a specialization of services have gone on, the importance of the services has reached a point where it overpasses the impact of wages and land rent as far as the location choice of the industry firms is concerned. Thus the post-modern city where industry and services are melted (usually in clusters) has appeared around the 1980's.

Note that to understand completely the last regime, the location choice of the services should be made entirely endogeneous and without constraints. This could be done in a further paper and should deliver not three but four regimes. The two first may still exist but the third might split into two different regimes. This third stage is characterized by the link between services and industry: what causes the re-agglomeration of the firms is not the geographical center of the city (wage curve) rather it is the proximity to the CBD. So if the services were to move into the periphery when the IC relocates there, the final regime would be characterized by an agglomeration of the IC around a peripheral specialized CBD. What Scott [27] witnesses in L.A. could be of such form.

Finally, note that we have remained here in the case of a unique city. While there is an externalization and a specialization of the services there is also a specialization of the cities within an economic region as Fujita an Hamaguchi [10] clearly worked it out. The primacy city would be more services oriented then the other cities and would also develop specialized services while the others could not. Hence, within a single economic region there could be different types of organization for cities of higher and lower ranks. For example, the central agglomeration would follow the post-modern regime of organization, with the growth of clusters around the historical CBD, while more peripheral cities would still be modern cities with services in the center and IC located around it. Thus, and apart from differences in size, this could explain why there are different types of urban organisation in the same urban region depending on the rank of the city in the regional hierarchy.

6 Appendix 1

When the preference for variety of the industry changes, its production

$$Y = L^{\frac{\sigma+\lambda-1}{\sigma-1}} (1-\lambda)^{1-\lambda} \left(\frac{2\lambda}{\sigma}\right)^{\frac{\sigma\lambda}{\sigma-1}} \left(\frac{1}{k}\right)^{\frac{\lambda}{\sigma-1}} (\sigma-1)^{\lambda}$$

changes too, and we reach

$$\frac{\partial Y}{\partial \sigma} = e^{\lambda \ln(\sigma-1)} \frac{e^{\frac{\sigma+\lambda-1}{\sigma-1}\ln L} e^{(1-\lambda)\ln(1-\lambda)} e^{\frac{\sigma\lambda}{\sigma-1}\ln 2\lambda}}{e^{\frac{\sigma\lambda}{\sigma-1}\ln\sigma} e^{\frac{\lambda}{\sigma-1}\ln k}} \frac{\lambda}{(\sigma-1)^2} \ln \frac{\sigma k}{2\lambda L}.$$

The sign of the preceding expression can then be determined with

$$\begin{aligned} \frac{\partial Y}{\partial \sigma} &> 0 \Leftrightarrow \ln \frac{\sigma k}{2\lambda L} > 0 \\ &\Leftrightarrow \sigma > \frac{2\lambda L}{k}. \end{aligned}$$

As long as $\sigma < \frac{2\lambda L}{k}$ (i.e.: a very high substituability of the services) a decrease in σ (the preference for variety increases) will raise the production of the city. It is only for very high (and mostly unlikely) values of σ ($\sigma > \frac{2\lambda L}{k}$) that an increase in the specialization (decrease in σ) is about to decrease the production of the city.

This case never happens because, with

$$N = \frac{2}{\sigma k} \frac{\lambda T(0) Y}{W(0)} = \frac{2\lambda}{\sigma k} L$$

we get

$$\frac{\partial Y}{\partial \sigma} < 0 \Leftrightarrow N > 1.$$

Hence, as long as there is a positive number of firms (that is to say as soon as the city exists...) an increase in the preference for variety will always have a positive effect on the production of the city. In order to understand the effect of a greater specialization on the structure of the services, note that

$$\frac{\partial N}{\partial \sigma} = -\frac{2\lambda}{\sigma^2 k} L < 0 \quad \text{et} \quad \frac{\partial \bar{q}}{\partial \sigma} = k > 0.$$

Then, if the services are more and more specialized, there will be both an increase in the number of services firms and a decrease in their size. And when the specialization increases, the number of variety effect gets more and more important compared to the size effect.

7 Appendix 2

When the industry externalizes its production, the production

$$Y = e^{\lambda \ln(\sigma-1)} \frac{e^{\frac{\sigma+\lambda-1}{\sigma-1} \ln L} e^{(1-\lambda)\ln(1-\lambda)} e^{\frac{\sigma\lambda}{\sigma-1} \ln 2\lambda}}{e^{\frac{\sigma\lambda}{\sigma-1} \ln \sigma} e^{\frac{\lambda}{\sigma-1} \ln k}},$$

the production evolves as follows:

$$\begin{aligned} \frac{\partial Y}{\partial \lambda} &= \frac{e^{\lambda \ln(\sigma-1)}}{\sigma - 1} \frac{e^{\frac{\sigma+\lambda-1}{\sigma-1} \ln L} e^{(1-\lambda)\ln(1-\lambda)} e^{\frac{\sigma\lambda}{\sigma-1} \ln 2\lambda}}{e^{\frac{\sigma\lambda}{\sigma-1} \ln \sigma} e^{\frac{\lambda}{\sigma-1} \ln k}} \\ &\quad * \left(\sigma \ln \frac{2(\sigma-1)\lambda}{\sigma(1-\lambda)} + \ln \frac{(1-\lambda) L}{(\sigma-1) k} + 1 - \frac{\sigma}{2} \right). \end{aligned}$$

Hence,

$$\frac{\partial Y}{\partial \lambda} > 0 \Leftrightarrow \sigma \ln \frac{2(\sigma-1)\lambda}{\sigma(1-\lambda)} + \ln \frac{(1-\lambda)L}{(\sigma-1)k} + 1 - \frac{\sigma}{2} > 0.$$

For most of the realistic cases, this is verified. $\frac{\partial Y}{\partial \lambda}$ is negative only for very low values of λ when the economy of the city is quite entirely dedicated to industry (fig. 8).

<Insert Figure 8>

The evolution of the production regarding the externalization , $\frac{\partial Y}{\partial \lambda}$, is always increasing as we see it on Figure 1 and it is negative only for low values of λ and very high values of σ.

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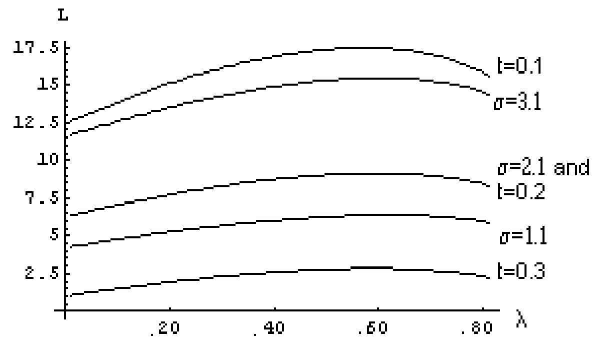

Figure 1: Size of the city, transportation cost and specialization of services

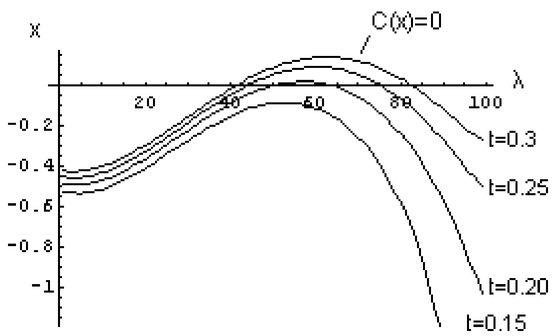

Figure 2: Evolution of the profit due to a relocation ($\sigma = 2$)

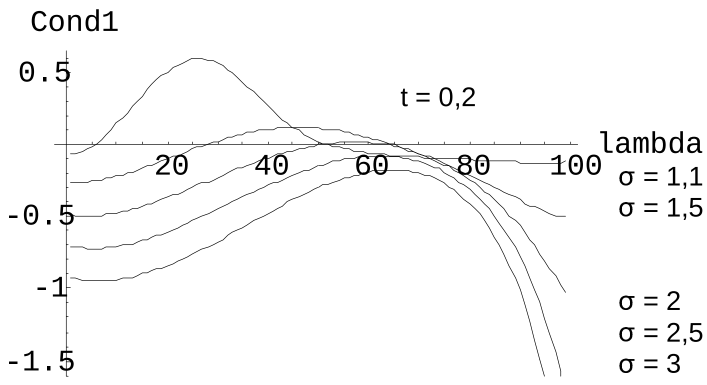

Figure 3: Evolution of the profit due to a relocation ($t = 0.2$)

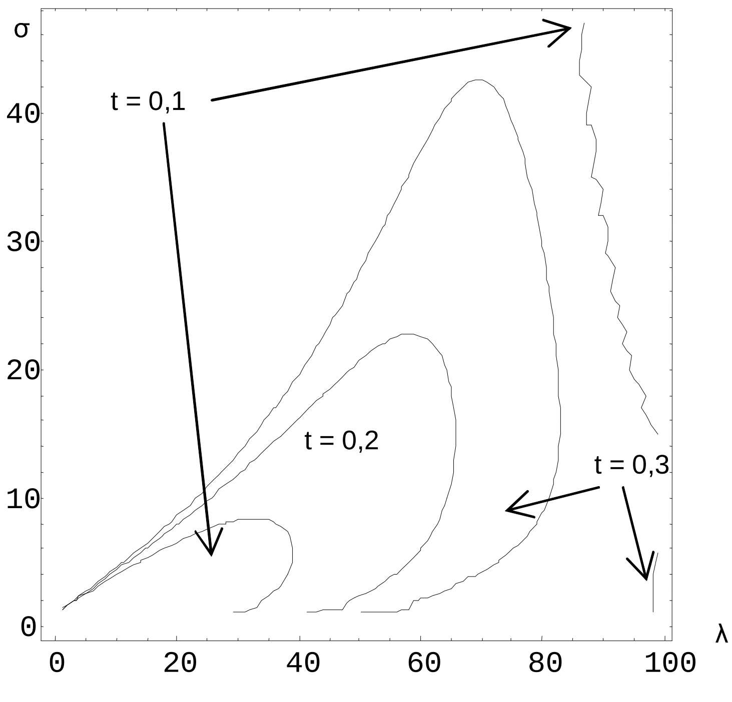

Figure 4: Ranges of parameters that clear the condition of existence of a monocentric city

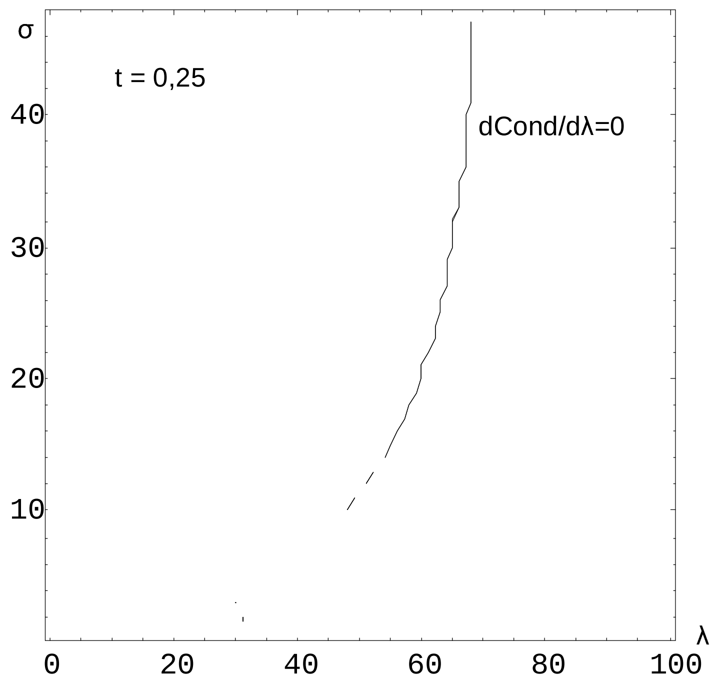

Figure 5: Maximum of the condition regarding externalization

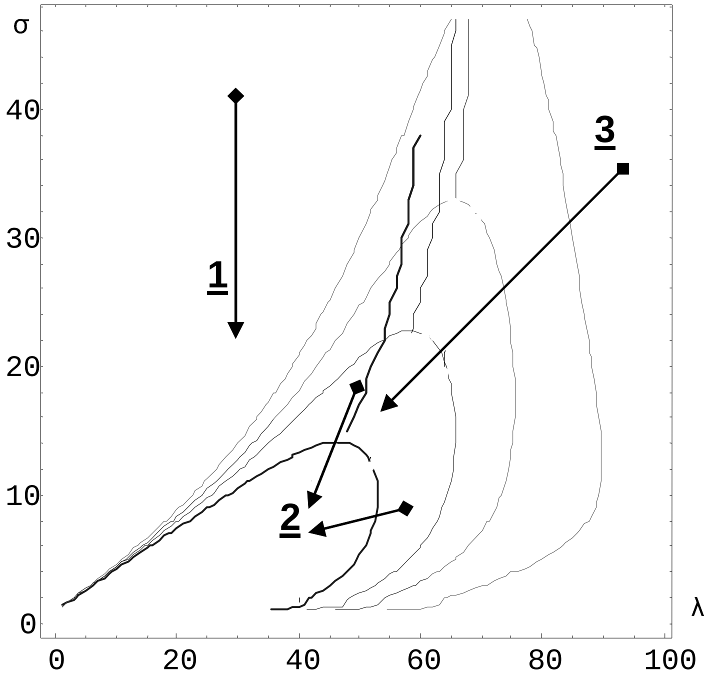

Figure 6: Effect of a decrease in the transportation cost

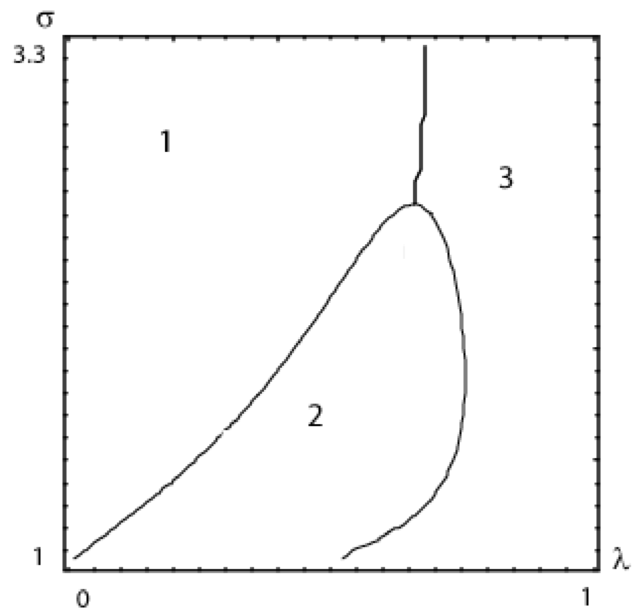

Figure 7: Three regimes of cities

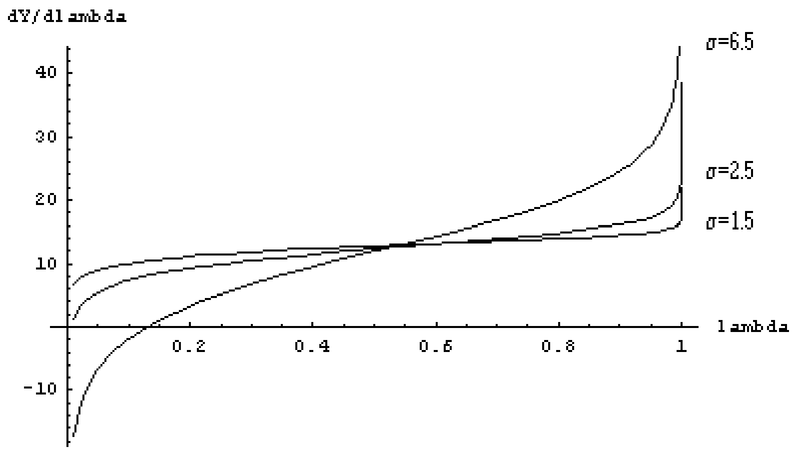

Figure 8: Production and externalization of the industry